Decomposing for Area and Perimeter

5^{th} Grade CST: NS1.4

13) Which of the following shows the number 60 factored into prime numbers?

A) 2×30

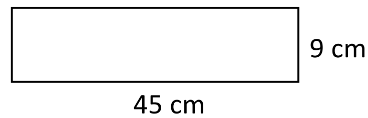
B) 3×20

C) $2 \times 3 \times 10$

D) $2 \times 2 \times 3 \times 5$

4^{th} Grade CST: AF1.4

49) Which equation below represents the area *(A)* of the rectangle in square centimeters?



A) $45 = A \times 9$

B) $A = 45 \times 9$

C) $A = (2 \times 45) + (2 \times 9)$

D) $45 = (2 \times A) + (2 \times 9)$

What is the perimeter of this rectangle? Give two other examples of rectangles that have the same perimeter, but different areas.

5^{th} Grade CST: MG1.4

65) A store has a rectangular parking lot that is 100 feet by 120 feet. What is the perimeter of the parking lot?

A) 220 feet

B) 440 feet

C) 1200 square feet

D) 12,000 square feet

Other

How many rectangles have equal perimeters and areas?

Objective: Students will use decomposing to find different dimensions for the given perimeter or area of rectangles.

Answer Key

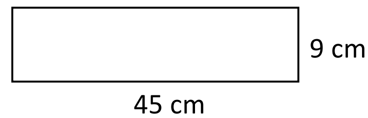
5th Grade CST: NS1.4

13) Which of the following shows the number 60 factored into prime numbers?

D) $2 \times 2 \times 3 \times 5$

4th Grade CST: AF1.4

49) Which equation below represents the area *(A)* of the rectangle in square centimeters?



B) $A = 45 \times 9$

What is the perimeter of this rectangle? Give two other examples of rectangles that have the same perimeter, but different areas.

$$P = 45\text{ cm} + 45\text{ cm} + 9\text{ cm} + 9\text{ cm}$$
$$P = 108\text{ cm}$$

Some examples of other rectangles: 1 cm x 53 cm, 2 cm x 52 cm, 3 cm x 51 cm

5th Grade CST: MG1.4

65) A store has a rectangular parking lot that is 100 feet by 120 feet. What is the perimeter of the parking lot?

B) 440 feet

Other

Two rectangles:

$$\text{Rectangle 1: } 6\text{ units} \times 3\text{ units}$$

$$\text{Rectangle 2: } 4\text{ units} \times 4\text{ units}$$

Decomposing for Area and Perimeter

Level: 4^{th} Grade

Measurement and Geometry Standard Set 1.0	
Students understand perimeter and area.	
4MG1.2	Recognize that rectangles that have the same area can have different perimeters.
4MG1.3	Understand that rectangles that have the same perimeter can have different areas.
4MG1.4	Understand and use formulas to solve problems involving perimeters and areas of rectangles and squares. Use those formulas to find the areas of more complex figures by dividing the figures into basic shapes.

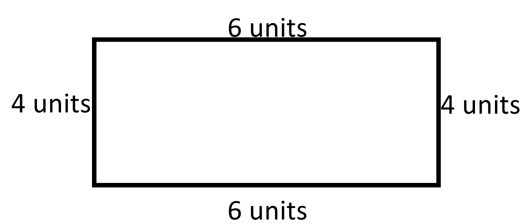
Measurement and Geometry Standard Set 3.0	
Students demonstrate an understanding of plane and solid geometric objects and use this knowledge to show relationships and solve problems.	
4MG3.3	Identify congruent figures.

Number Sense Standard Set 4.0	
Students know how to factor small whole numbers.	
4NS4.1	Understand that many whole numbers break down in different ways.
4NS4.2*	Know that numbers such as 2, 3, 5, 7 and 11 do not have any factors except 1 and themselves and that such numbers are called ***prime numbers***.

Objective: Students will decompose by either factors or addends to find different dimensions of rectangles when given the area or perimeter.

Vocabulary-

Perimeter: peri- a Greek prefix meaning *around*; linear measurement of all sides of a polygon, one dimensional measurement *(use ruler, unit cubes to reinforce linear dimension).*

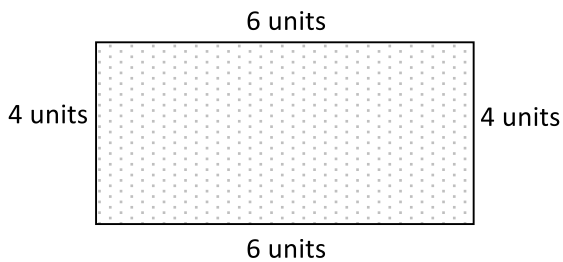


The perimeter of this rectangle is $6\text{ units} + 6\text{ units} + 4\text{ units} + 4\text{ units} = 20\text{ units}$.

***Note**: Focus on using addition **only** before moving to the use of the perimeter formula.*

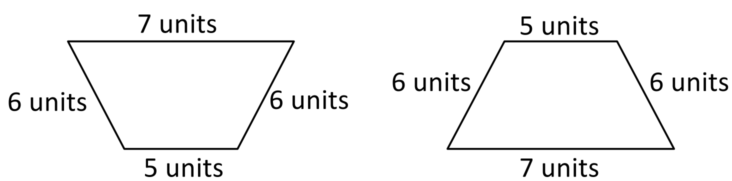
Area: the two-dimensional measurement of space within a perimeter (or circumference). Area of a rectangle can be determined by multiplying its base by its height *(use multiplication chart or arrays to reinforce two dimensions).*

Note: *Discuss with students 'base x height' vs 'length x width' (which will be seen in texts and on assessments). 'Base x height' is the preferred terminology for area; this differentiates between finding area and finding volume, when the formula 'base x height x width' is used.*



The area of this rectangle is $6\text{ units}\times 4\text{ units} = 24\text{ units squared, or }24\text{ square units.}$

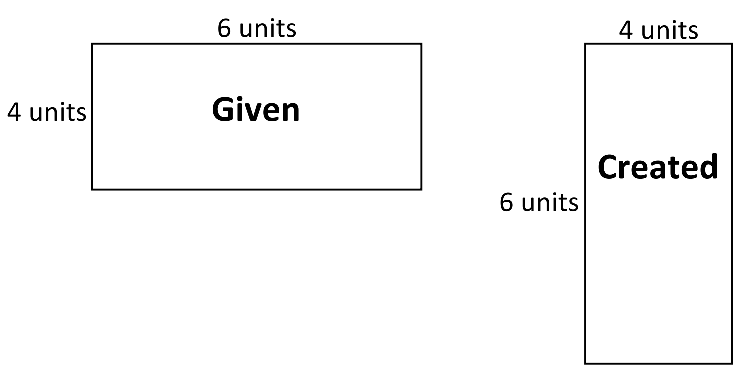
Congruent ($\cong$): two geometric figures that have the same size (=) and shape (~).



These two trapezoids are congruent $\cong$.

<u>Concept Development: Area</u>

Common Errors: When given a rectangle and asked to create another with different perimeters but the same area, many students will create a congruent rectangle.

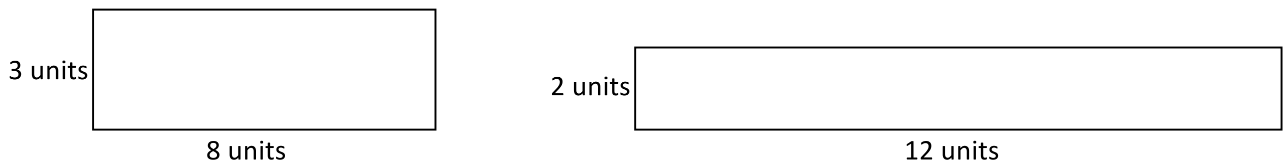


Since the rectangles are congruent, they will have the same area **and** the same perimeter, which means the dimensions are **not** different.

Perimeter of given rectangle: 6 units + 6 units + 4 units + 4 units = 20 units.

Perimeter of created rectangle: 4 units + 4 units + 6 units + 6 units = 20 units.

Some students ***will*** find other dimensions, but their findings can be limited.

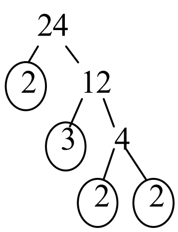


4MG1.2: Recognize that rectangles that have the same area can have different perimeters.

Demonstrate/model using graph paper, and have the students do the same with the You Tries.

"Our task is to find all possible (whole number) dimensions for a rectangle with an area of 24 square units. We will do this by factoring."

"First, decompose the given area (24 square units) to prime, and then list all the factors in numeric order; since 1 is a factor of every number, be sure to include it, too."



24: 1, 2, 2, 2, 3

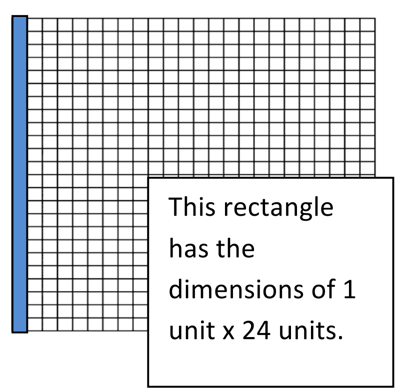
"Since the area is the product of the base and height, we can use the factors listed to make all the different base and height combinations for a rectangle with an area of 24 square units. Start with a base of 1, and then multiply all the other factors listed to find the height for a rectangle with an area of 24 square units. "

$$(1)(2 \cdot 2 \cdot 2 \cdot 3) = 24$$

$$\text{Area } (1 \text{ unit})(24 \text{ unit}) = 24 \text{ square units}$$

$$P = 1 \text{ unit} + 1 \text{ unit} + 24 \text{ units} + 24 \text{ units}$$
$$P = 50 \text{ units}$$

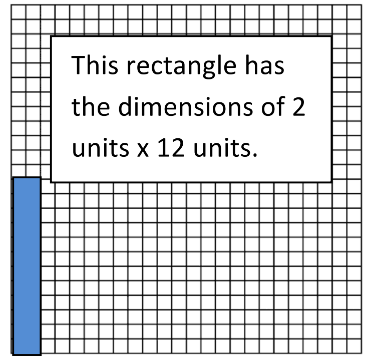
"The area is 24 square units, and the perimeter is 50 units."



"Now, keeping numeric order, see if there can be a base of 2 for this rectangle. Since we have a factor of 2 we can use that for the base. Now multiply the other factors together to get the other dimension (the height) for a rectangle with an area of 24 square units."

$$(2)(1 \cdot 2 \cdot 2 \cdot 3) = 24$$

$$\text{Area } (2 \text{ units})(12 \text{ units}) = 24 \text{ square units}$$



$$P = 2 \text{ units} + 2 \text{ units} + 12 \text{ units} + 12 \text{ units}$$

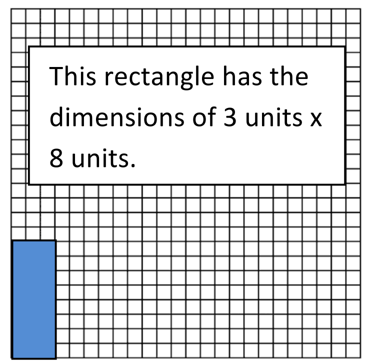
$$P = 28 \text{ units}$$

"The area is 24 square units, and the perimeter is 28 units."

"Now, keeping numeric order again, let's see…Can there be a base of 3?" [Yes] "We'll multiply the other factors together to find the other dimension (the height) for
24 square units."

$$(3)(1 \cdot 2 \cdot 2 \cdot 2) = 24$$

$$\text{Area } (3 \text{ units})(8 \text{ units}) = 24 \text{ square units}$$



$$P = 3 \text{ units} + 3 \text{ units} + 8 \text{ units} + 8 \text{ units}$$

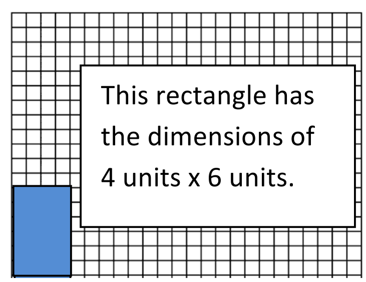
$$P = 22 \text{ units}$$

"The area is 24 square units, and the perimeter is 22 units."

(Repeat these steps, keeping numeric order.) "Now, staying in order
factors together to get a base of 4?" [Yes, 2 x 2] "Now we multiply the other factors together to get the height for our rectangle."

$$(2 \cdot 2)(1 \cdot 2 \cdot 3) = 24$$

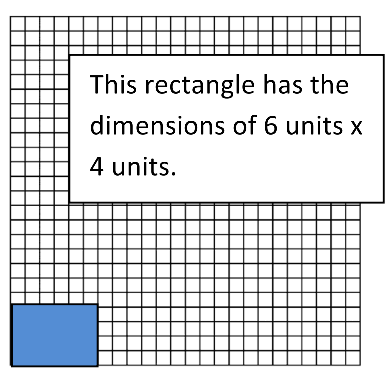
$$\text{Area } (4 \text{ units})(6 \text{ units}) = 24 \text{ square units}$$



$$P = 4 \text{ units} + 4 \text{ units} + 6 \text{ units} + 6 \text{ units}$$

$$P = 20 \text{ units}$$

"What is the area of this rectangle?" [24 square units.] "What is the perimeter of this rectangle?" [20 units.]



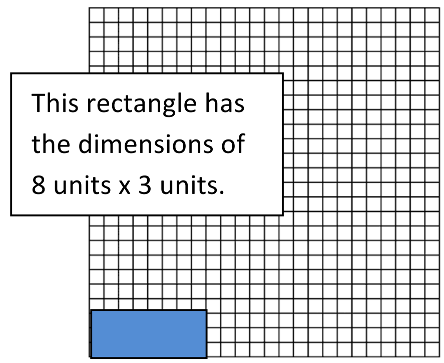
"Now for the next rectangle, can there be a base of 5?" [No, 5 is not a factor of 24] "So we'll check if there can be a base of 6…Are there any factors that we can multiply to create a base of 6?” [Yes, 2 x 3] “Now multiply the rest of the factors together to get the height for a rectangle with an area of 24 square units.”

$$(2 \cdot 3)(1 \cdot 2 \cdot 2) = 24$$

$$\text{Area } (6 \text{ units})(4 \text{ units}) = 24 \text{ square units}$$

$$P = 6 \text{ units} + 6 \text{ units} + 4 \text{ units} + 4 \text{ units}$$
$$P = 20 \text{ units}$$

“What is the area of this rectangle?” [24 square units.] “What is the perimeter of this rectangle?” [20 units.]



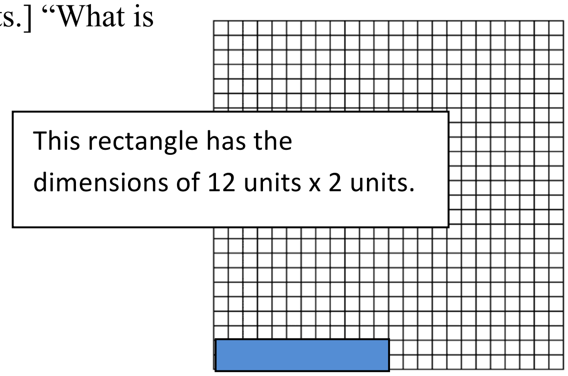
Continue following these orderly steps until all possible rectangle dimensions are found—see rectangles below.

$$(2 \cdot 2 \cdot 2)(1 \cdot 3) = 24$$

$$\text{Area } (8 \text{ units})(3 \text{ units}) = 24 \text{ square units}$$

$$P = 8 \text{ units} + 8 \text{ units} + 3 \text{ units} + 3 \text{ units}$$
$$P = 22 \text{ units}$$

“What is the area of this rectangle?” [24 square units.] “What is the perimeter of this rectangle?” [22 units.]



$$(2 \cdot 2 \cdot 3)(1 \cdot 2) = 24$$

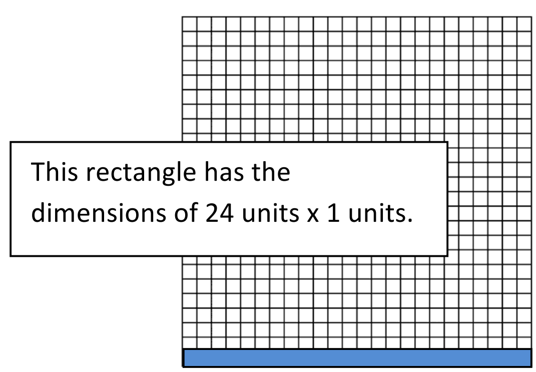
$$\text{Area } (12 \text{ units})(2 \text{ units}) = 24 \text{ square units}$$

$$P = 12 \text{ units} + 12 \text{ units} + 2 \text{ units} + 2 \text{ units}$$
$$P = 28 \text{ units}$$

“Area?” [24 square units.] “Perimeter?” [28 units.]

$$(2 \cdot 2 \cdot 2 \cdot 3)(1) = 24$$

$$\text{Area } (24 \text{ units})(1 \text{ unit}) = 24 \text{ square units}$$



$$P = 24 \text{ units} + 24 \text{ units} + 1 \text{ unit} + 1 \text{ unit}$$
$$P = 50 \text{ units}$$

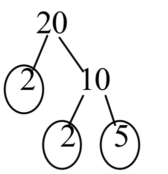
“Area?” [24 square units.] “Perimeter?” [50 units.]

“Since we have a factor of 1 for our height, kept numeric order, and used all factors in each rectangle created, we have found all possible dimensions for a rectangle with 24 square units.”

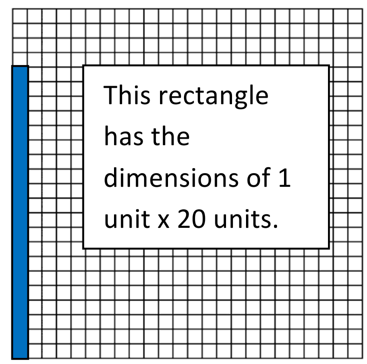
Show all rectangles side by side so that students can easily see the changes in dimensions and make comparisons.

You Try

Use factoring to find all possible dimensions for a rectangle with an area of 20 square units.



20: 1, 2, 2, 5



$$(1)(2 \cdot 2 \cdot 5) = 20$$

$$\text{Area } (1 \text{ unit})(20 \text{ units}) = 20 \text{ square units}$$

$$P = 1 \text{ unit} + 1 \text{ unit} + 20 \text{ units} + 20 \text{ units}$$

$$P = 42 \text{ units}$$

$$A = 20 \text{ square units}, P = 42 \text{ units}$$

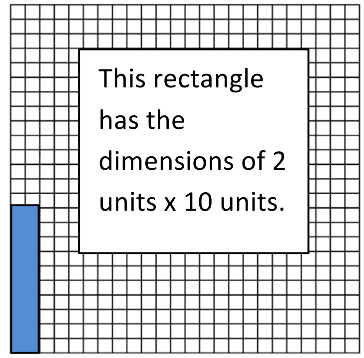
$$(2)(1 \cdot 2 \cdot 5) = 20$$

$$\text{Area } (2 \text{ units})(10 \text{ units}) = 20 \text{ square units}$$

$$P = 2 \text{ units} + 2 \text{ units} + 10 \text{ units} + 10 \text{ units}$$

$$P = 24 \text{ units}$$

$$A = 20 \text{ square units}, P = 24 \text{ units}$$



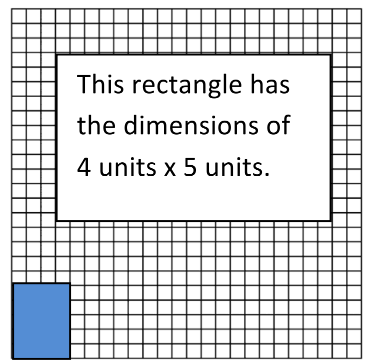
$$(2 \cdot 2)(1 \cdot 5) = 20$$

$$\text{Area } (4 \text{ units})(5 \text{ units}) = 20 \text{ square units}$$

$$P = 4 \text{ units} + 4 \text{ units} + 5 \text{ units} + 5 \text{ units}$$

$$P = 18 \text{ units}$$

$$A = 20 \text{ square units, } P = 18 \text{ units}$$

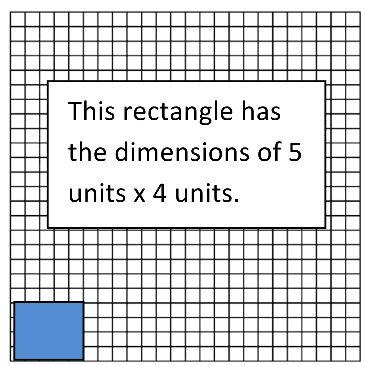


$$(5)(1 \cdot 2 \cdot 2) = 20$$

$$\text{Area } (5 \text{ units})(4 \text{ units}) = 20 \text{ square units}$$

$$P = 5 \text{ units} + 5 \text{ units} + 4 \text{ units} + 4 \text{ units}$$

$$P = 18 \text{ units}$$



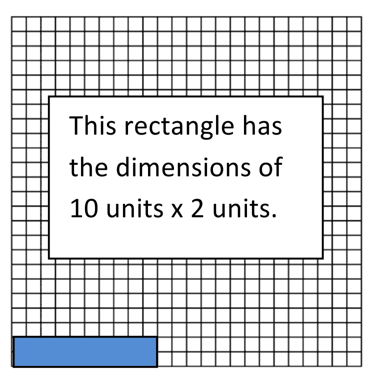
$$A = 20 \text{ square units, } P = 18 \text{ units}$$

$$(2 \cdot 5)(1 \cdot 2) = 20$$

$$\text{Area } (10 \text{ units})(2 \text{ units}) = 20 \text{ square units}$$

$$P = 10 \text{ units} + 10 \text{ units} + 2 \text{ units} + 2 \text{ units}$$

$$P = 24 \text{ units}$$



$$A = 20 \text{ square units, } P = 24 \text{ units}$$

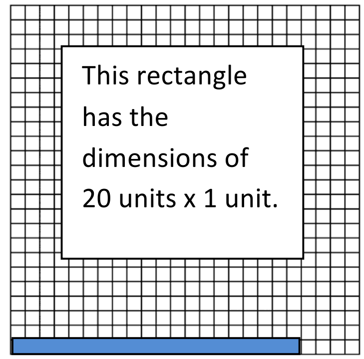
$$(2 \cdot 2 \cdot 5)(1) = 20$$

$$\text{Area } (20 \text{ units})(1 \text{ unit}) = 20 \text{ square units}$$

$$P = 20 \text{ units} + 20 \text{ units} + 1 \text{ unit} + 1 \text{ unit}$$

$$P = 42 \text{ units}$$

$$A = 20 \text{ square units, } P = 42 \text{ units}$$



Frontloading: "Review your rectangles and discuss with a partner how to find the perimeter." [Add, then double/multiply by 2] *Leads into using the distributive property and formula.*

Concept Development: Perimeter

Since we have reinforced using addition for finding the perimeter, we now can decompose by addends to find rectangles with the same perimeter but different areas.

Standard 4MG1.3: Rectangles with the same perimeter have different areas.

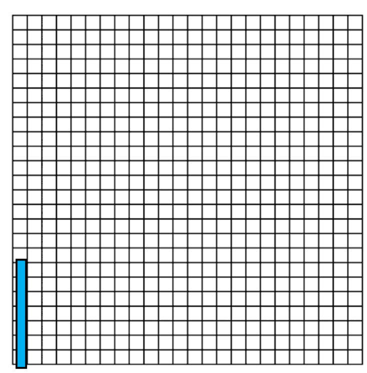
"We can use what we know about rectangles and decompose by addends to prove this standard. When given that Rectangle 1 has a perimeter of 16 units, our task is to find all the possible dimensions of a rectangle with a perimeter of 16 units."

"What if we started decomposing 16 to 2 +14? Discuss with your neighbor what the sides of the rectangle would be." *Allow time for discussion and drawing their ideas, then ask student volunteers to share their thinking.* [Dimensions would be 1 x 7 because 2 + 14 only shows two sides, so each number needs to be divided in half.]

$$P_1 = 16 \text{ units}$$

$$P_1 = 2 \text{ units} + 14 \text{ units}$$

$$P_1 = 1 \text{ unit} + 1 \text{ unit} + 7 \text{ units} + 7 \text{ units}$$



This rectangle has the dimensions of 1 unit x 7 units.

$$A_1 = 1 \text{ unit} \cdot 7 \text{ units}$$

$$A_1 = 7 \text{ units squared}$$

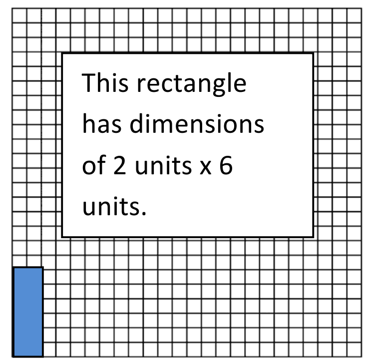
"Remember to keep numeric order and you will be able to find all the (whole number) different dimensions for a rectangle with a perimeter of 16."

Ask students to discuss amongst themselves if it is possible for a rectangle to have fractions/decimals as its dimensions; support and give examples. [Yes; sample response: 4.25 inches for the base and 5.5 inches for the height.] Confirm this with students, and share that since the standard for 4th grade regards whole numbers, we will only focus on using whole numbers while we are developing the concept.

"We started decomposing the perimeter of 16 units in our first rectangle as 2 + 14. Remember that we want to keep order, and use only whole numbers as the base and height, which means that we should decompose the perimeter of our second rectangle as 4 + 12; the next step is to decompose each of these addends in half, as 4 + 12 only represents two sides of the rectangle."

$P_2 = 16 \text{ units}$
$P_2 = 4 \text{ units} + 12 \text{ units}$
$P_2 = 2 \text{ units} + 2 \text{ units} + 6 \text{ units} + 6 \text{ units}$

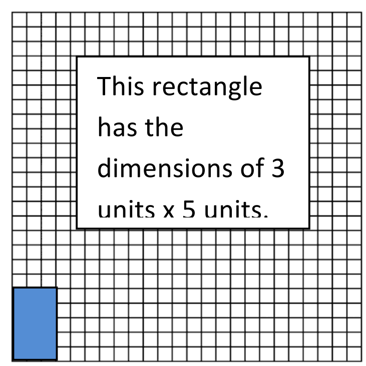
$A_2 = 2 \text{ units} \cdot 6 \text{ units}$
$A_2 = 12 \text{ units squared}$



"Let's decompose the dimensions for rectangle 3, following the same steps for rectangles 1 and 2. We'll start the decomposing with a base of 6 and height of 10, then take half of each addend."

$P_3 = 16 \text{ units}$
$P_3 = 6 \text{ units} + 10 \text{ units}$
$P_3 = 3 \text{ units} + 3 \text{ units} + 5 \text{ units} + 5 \text{ units}$

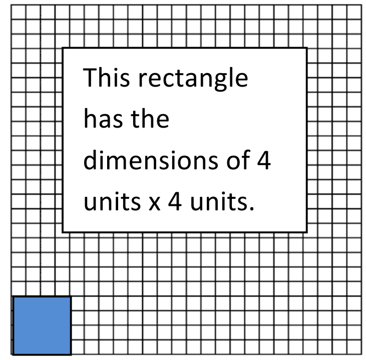
$A_3 = 3 \text{ units} \cdot 5 \text{ units}$
$A_3 = 15 \text{ units squared}$



"Now we'll follow the same steps to find the dimensions for rectangle 4. What base should we start with?" [8, then divide in half]

$P_4 = 16 \text{ units}$
$P_4 = 8 \text{ units} + 8 \text{ units}$
$P_4 = 4 \text{ units} + 4 \text{ units} + 4 \text{ units} + 4 \text{ units}$

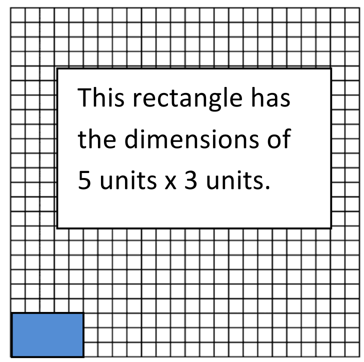
$A_4 = 4 \text{ units} \cdot 4 \text{ units}$
$A_4 = 16 \text{ units squared}$



"How should we start decomposing for the fifth rectangle?" [10 + 6, then decompose each addend in half.]

$P_5 = 16 \text{ units}$
$P_5 = 10 \text{ units} + 6 \text{ units}$
$P_5 = 5 \text{ units} + 5 \text{ units} + 3 \text{ units} + 3 \text{ units}$

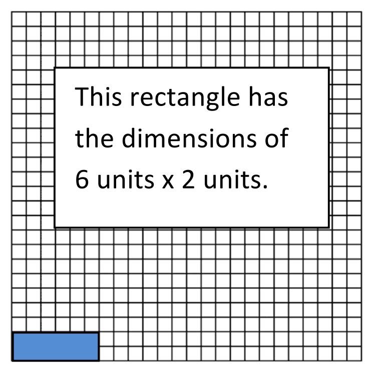
$A_5 = 5 \text{ units} \cdot 3 \text{ units}$
$A_5 = 15 \text{ units squared}$



"How will we start decomposing for the sixth rectangle?" [12 + 4, then decompose each addend in half.]

$P_6 = 16 \text{ units}$
$P_6 = 12 \text{ units} + 4 \text{ units}$
$P_6 = 6 \text{ units} + 6 \text{ units} + 2 \text{ units} + 2 \text{ units}$

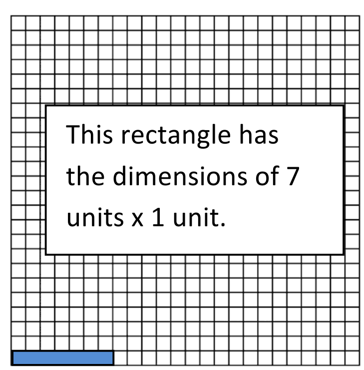
$A_6 = 6 \text{ units} \cdot 2 \text{ units}$
$A_6 = 12 \text{ units squared}$



"How do we start the seventh rectangle?" [14 + 2, then decompose each addend in half.]

$P_7 = 16 \text{ units}$
$P_7 = 14 \text{ units} + 2 \text{ units}$
$P_7 = 7 \text{ units} + 7 \text{ units} + 1 \text{ unit} + 1 \text{ unit}$

$A_7 = 7 \text{ units} \cdot 1 \text{ unit}$
$A_7 = 7 \text{ units squared}$



Have students discuss the relationship and the patterns they see in the rectangles; make and display posters. While working in groups of 2 or 3, challenge them to come up with different ways to write the perimeter and share their ideas.

[P = 2(b) + 2(h); P = 2 (b + h)]

Discuss the perimeter formula. Have students use the perimeter formula for all the rectangles.

Perimeter Solutions:

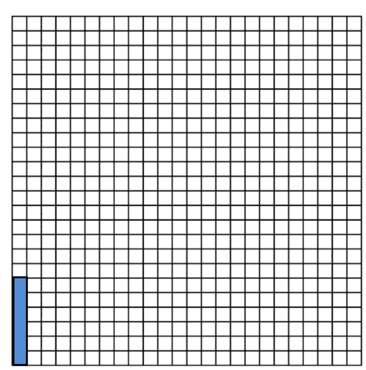
$P_1 = 2(1\text{ unit}) + 2(7\text{ units})$ $= 2\text{ units} + 14\text{ units}$ $= 16\text{ units}$	$P_2 = 2(2\text{ units}) + 2(6\text{ units})$ $= 4\text{ units} + 12\text{ units}$ $= 16\text{ units}$	$P_3 = 2(3\text{ units}) + 2(5\text{ units})$ $= 6\text{ units} + 10\text{ units}$ $= 16\text{ units}$
$P_4 = 2(4\text{ units}) + 2(4\text{ units})$ $= 8\text{ units} + 8\text{ units}$ $= 16\text{ units}$	$P_5 = 2(5\text{ units}) + 2(3\text{ units})$ $= 10\text{ units} + 6\text{ units}$ $= 16\text{ units}$	$P_6 = 2(6\text{ units}) + 2(2\text{ units})$ $= 12\text{ units} + 4\text{ units}$ $= 16\text{ units}$
$P_7 = 2(7\text{ units}) + 2(1\text{ unit})$ $= 14\text{ units} + 2\text{ units}$ $= 16\text{ units}$		

You Try

Decompose by addends to find all possible dimensions for a rectangle with a perimeter of 14 units. Then, use the perimeter formula to solve.

$P_1 = 14\text{ units}$
$P_1 = 2\text{ units} + 12\text{ units}$
$P_1 = 1\text{ unit} + 1\text{ unit} + 6\text{ units} + 6\text{ units}$

$A_1 = 1\text{ unit} \cdot 6\text{ units}$
$A_1 = 6\text{ units squared}$



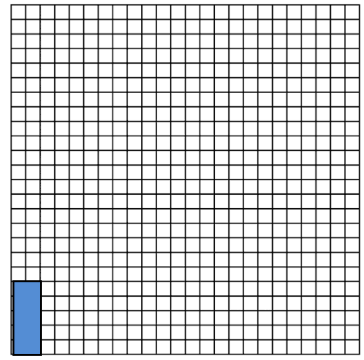
$P_2 = 14 \text{ units}$

$P_2 = 4 \text{ units} + 10 \text{ units}$

$P_2 = 2 \text{ units} + 2 \text{ units} + 5 \text{ units} + 5 \text{ units}$

$A_2 = 2 \text{ units} \cdot 5 \text{ units}$

$A_2 = 10 \text{ units squared}$



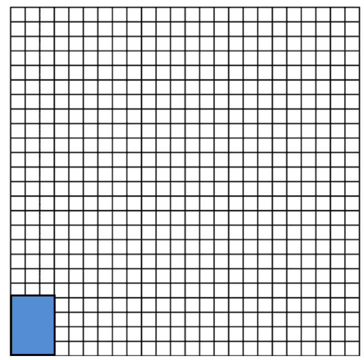
$P_3 = 14 \text{ units}$

$P_3 = 6 \text{ units} + 8 \text{ units}$

$P_3 = 3 \text{ units} + 3 \text{ units} + 4 \text{ units} + 4 \text{ units}$

$A_3 = 3 \text{ units} \cdot 4 \text{ units}$

$A_3 = 12 \text{ units squared}$



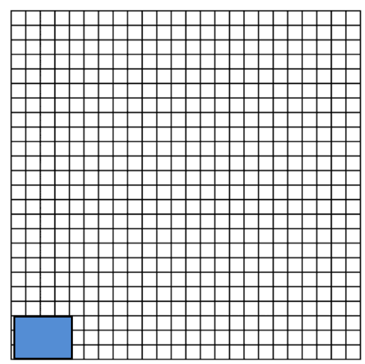
$P_4 = 14 \text{ units}$

$P_4 = 8 \text{ units} + 6 \text{ units}$

$P_4 = 4 \text{ units} + 4 \text{ units} + 3 \text{ units} + 3 \text{ units}$

$A_4 = 4 \text{ units} \cdot 3 \text{ units}$

$A_4 = 12 \text{ units squared}$



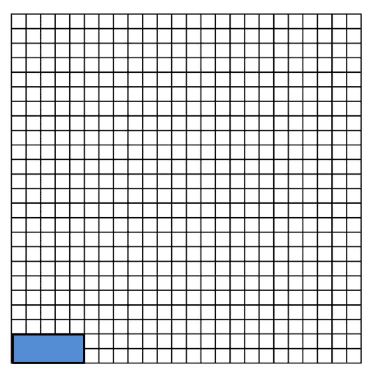
$P_5 = 14 \text{ units}$

$P_5 = 10 \text{ units} + 4 \text{ units}$

$P_5 = 5 \text{ units} + 5 \text{ units} + 2 \text{ units} + 2 \text{ units}$

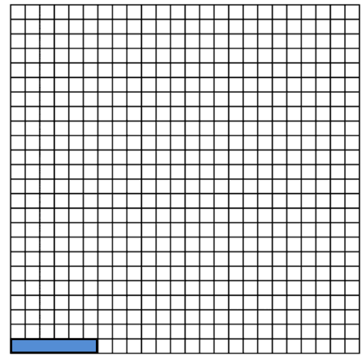
$A_5 = 5 \text{ units} \cdot 2 \text{ units}$

$A_5 = 10 \text{ units squared}$



$P_6 = 14 \text{ units}$
$P_6 = 12 \text{ units} + 2 \text{ units}$
$P_6 = 6 \text{ units} + 6 \text{ units} + 1 \text{ unit} + 1 \text{ unit}$

$A_6 = 6 \text{ units} \cdot 1 \text{ unit}$
$A_6 = 6 \text{ units squared}$



Perimeter Solutions

$P_1 = 2(1 \text{ unit}) + 2(6 \text{ units})$ $= 2 \text{ units} + 12 \text{ units}$ $= 14 \text{ units}$	$P_2 = 2(2 \text{ units}) + 2(5 \text{ units})$ $= 4 \text{ units} + 10 \text{ units}$ $= 14 \text{ units}$	$P_3 = 2(3 \text{ units}) + 2(4 \text{ units})$ $= 6 \text{ units} + 8 \text{ units}$ $= 14 \text{ units}$
$P_4 = 2(4 \text{ units}) + 2(3 \text{ units})$ $= 8 \text{ units} + 6 \text{ units}$ $= 14 \text{ units}$	$P_5 = 2(5 \text{ units}) + 2(2 \text{ units})$ $= 10 \text{ units} + 4 \text{ units}$ $= 14 \text{ units}$	$P_6 = 2(6 \text{ units}) + 2(1 \text{ unit})$ $= 12 \text{ units} + 2 \text{ units}$ $= 14 \text{ units}$

Concluding Notes: While the concepts and skills are not too challenging, and the use of formulas is not difficult for students to grasp, the opportunity to decompose numbers and manipulate them into given areas and perimeters helps to develop a strong number sense foundation.